Parameter Identification of Discrete-Time Series Models for Structural Response Prediction

S. M. Batill* and J. J. Hollkamp†
University of Notre Dame, Notre Dame, Indiana

An experimental study of a time-domain method for modeling and parameter identification of structural systems is presented. Models are developed that can be used to predict the transient response of multiple-degree-of-freedom systems subjected to arbitrary input. The linear, discrete-time transfer function is expressed in a form called the autoregressive moving average (ARMA) model. Although it has proven useful for a number of low-order systems, the identification of the ARMA model is often hampered by the sensitivity of parameter estimates to noise bias. A two-stage identification technique that uses overspecified models to estimate the poles of the ARMA model from system free-vibration response or time-domain impulse response data using a form of the principal eigenvectors method is presented. A least-squares algorithm is then used to identify the transfer-function zeros from forced response records. Experimental response data for both single and multiple degrees of freedom are used to evaluate the technique.

Nomenclature

a_i = autoregressive parameter
$A(z)$ = characteristic equation in the z plane
A_m = matrix of measured values
b = vector of moving average parameters
b_i = moving average parameter
$B(z)$ = moving average polynomial
c = vector of autoregressive parameters
c_i = autoregressive parameter
$C(z)$ = backward polynomial
d_m = vector of measured responses
e = vector of residuals
$e(k)$ = residual sequence
f = vector used in the principal eigenvectors method
f_s = sample rate
g_m = vector of measured responses
h = time interval
Q_m = matrix of measured values
s_i = pole in the Laplace domain
u_k = part of the singular value decomposition of a matrix
$u(k)$ = input sequence
$u_m(k)$ = measured input sequence
U_m = matrix of measured inputs
v_k = part of the singular value decomposition of a matrix
$y(k)$ = response sequence
$y_m(k)$ = measured response sequence
z = forward shift operator
z_i = pole in the z plane
$z_m(k)$ = calculated sequence for the second stage
z_m = vector of calculated sequence
θ = parameter vector
θ_{LS} = least-squares estimated parameter vector
σ_k = singular value of a matrix

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*Associate Professor, Aerospace and Mechanical Engineering. Associate Fellow AIAA.

†Research Assistant, Aerospace and Mechanical Engineering. Student Member AIAA.

Introduction

EXCESSIVE vibrations can damage or destroy a structure. Accurate transient dynamic response prediction is essential to avoid damage or degraded performance. During service, certain types of structures can assume many different "configurations": mass loadings change, structural properties can vary with temperature, localized component failure may occur. If one is concerned with the prediction of the dynamic response of critical components of a complete structure when subjected to a specific loading condition, then a single invariant structural model may not prove suitable. The many possible configurations that some structures assume during normal operation place limits on the usefulness of certain detailed analytic or numerical modeling procedures, such as finite-element analysis. It would be desirable to have techniques that would provide "on-line" models and would be suitable for the rapid and accurate prediction of transient dynamic response for a specific set of loading conditions.

Consider the problem of an airplane about to traverse a rough or damaged runway. Prediction of peak acceleration levels at several critical locations on the structure would be helpful in determining the speed and path the aircraft should take to minimize the risk of damage. A model relating the vibration (peak amplitude or acceleration) at a finite number of critical locations within the structure to the specific runway profile would be needed. However, each time the aircraft is used, its structural configuration may change, e.g., the payload or fuel loading may be modified. When the structural configuration changes, the predictive model must change. Or consider future large space structures that will be fabricated in orbit. Dynamic models will be necessary to control these structures. The varying structural characteristics during assembly would outdate the baseline analytical or modal models used to predict dynamic response.

In both examples, input to the structure is provided at specific locations and the system response may be necessary at only a finite number of positions on the structure. It would be desirable to identify the structural characteristics every time the structure is used. Ideally, digital time histories of the input and system output could be acquired and, using that information, the system model or transfer function could be derived or updated automatically and, thus, influence the operation of the system. The applications presented in this paper are not intended for system control and, therefore, the model development can be performed in a rapid batch process after all of the data have been acquired.

For linear systems, discrete-time transfer functions are well suited for automatic digital identification. One form of a discrete-time transfer function is the autoregressive moving average (ARMA) model. The basic ARMA model is the single-input/single-output (SISO) model, which has the form

$$y(k) = -a_1y(k-1) - a_2y(k-2) - \cdots - a_py(k-p) + b_0u(k) + b_1u(k-1) + \cdots + b_pu(k-p) \quad (1)$$

Applications of the ARMA model include digital filtering, substructure modeling,[1] spectral matching for probabilistic simulations[2,3] and vibration parameter extraction,[4] and many more in the controls field.

The ARMA model can be put in the discrete-time transfer-function form using shift operator notation

$$y(k) = \frac{b_0 + b_1z^{-1} + \cdots + b_pz^{-p}}{1 + a_1z^{-1} + \cdots + a_pz^{-p}}u(k) = \frac{B(z)}{A(z)}u(k) \quad (2)$$

where z is the forward shift operator. The roots of the $A(z)$ polynomial are the system poles, and the roots of the $B(z)$ polynomial are the zeros in the z plane. If there is no input, which is the case for free response, the ARMA model reduces to an autoregressive (AR) model. The roots of $A(z)$ or the poles in the z plane, z_i, can be mapped into the Laplace domain using

$$s_i = (1/h)\ell n(z_i) \quad (3)$$

where h is the uniform time increment between the discrete-time values. The natural frequencies and dampings can be extracted from the location of the poles in the s plane, s_i. An unstable pole in the s plane is mapped outside the unit circle in the z plane. The mapping provides a means of estimating the traditional structural quantities – frequency and damping – from the AR coefficients of an ARMA or AR model. This mapping is also used in statistical signal processing techniques such as the extended Prony's method and the Pisarenko method.[5]

There are advantages to the use of the ARMA models in the identification and prediction. The ARMA model is a minimum parameter model; therefore, it requires the least number of parameters to characterize the system. The SISO ARMA model's states are regressed discrete-time values of a single input and single output. The identification and prediction procedures also require the least number of measured quantities. Unfortunately, there are also certain disadvantages of ARMA models, which appear in the identification process.

The goal of the identification scheme presented in this paper is to estimate an ARMA model from experimental input and output time histories so that the model can be used to predict accurately the response to other inputs. Several factors influence the success of the parameter identification process. One of the most important factors is the selection of the order of the system, which has to be determined prior to parameter identification. Any continuous system obviously will be of infinite order, but, in practice, the order will be finite in a given frequency range. The frequency range will be limited by the discrete sampling frequency or possible data filtering. A model with an overspecified order may still yield adequate results, but a model with an underspecified order will not. Various methods to determine system order range from doubling the number of counted peaks in the frequency spectrum, the Akaike Information Criterion,[6] to a determination of the maximum possible rank of the free-response autocorrelation matrix.[7] Other factors in the parameter identification process are the selection of the sample frequency and use of lowpass filters. Finally, the most important factor is the selection of the identification algorithm.

This paper is concerned with the identification of ARMA models for structural response and will be limited to SISO applications. A classical identification scheme, the single-stage least-squares (SLS) algorithm, and a proposed modification, a two-stage least-squares (2LS) algorithm, are presented. Experimental results using the SLS and 2LS algorithms to estimate single-degree (SDOF) and multiple-degree-of-freedom (MDOF) system models are discussed. These results were achieved using actual experimental data, and the models were then used to predict transient response for these systems.

SLS Algorithm

When noise corrupts the measured input and output discrete-time histories, it is impossible to determine exactly the ARMA model parameters. An estimate of the parameters can be made by solving an overdetermined set of equations. If there are N data points in each time history, $N-p$ equations can be written, where p is the model order. The equations in matrix form are as follows:

$$A_m\theta = g_m + e \quad (4)$$

where

$$\theta = [a_1, a_2, \ldots, a_p, b_0, b_1, \ldots, b_p]^T \quad (5)$$

$$g_m = [y_m(p+1), y_m(p+2), \ldots, y_m(N)]^T \quad (6)$$

$$e = [e(p+1), e(p+2), \ldots, e(N)]^T \quad (7)$$

and

$$A_m = \begin{bmatrix} -y_m(p)\ldots -y_m(1) & u_m(p+1)\ldots u_m(1) \\ -y_m(p+1)\ldots -y_m(2) & u_m(p+2)\ldots u_m(2) \\ \vdots & \vdots \\ -y_m(N-1)\ldots -y_m(N-p) & u_m(N)\ldots u_m(N-p) \end{bmatrix} \quad (8)$$

where $e(k)$ is defined as the residual sequence, and the subscript m denotes a measured (and, therefore, noise-corrupted) quantity. Solving Eq. (4) by minimizing the sum of the squares of the residual, results in the estimated parameter vector

$$\theta_{LS} = (A_m{}^TA_m)^{-1}A_m{}^Tg_m \quad (9)$$

It is well known that the LS estimate is biased.[8,9] The sensitivity of the LS estimates to noise often makes the estimates unreliable.

There are many iterative algorithms that attempt to reduce the bias. The main algorithms are the generalized least-squares algorithm, the maximum likelihood method, and the instrumental variables method.[8] These algorithms have been successful to varying degrees, but the most effective and simplest method has been the order overspecified least-squares method. If p is the true order of the system, an overspecified model is oversized to order L, where L is greater than p. The overspecified LS method identifies a model with an oversized order using the LS algorithm. Overspecified least-squares methods have been used in conjunction with time-domain modal methods, such as Ibrahim's time-domain technique[10] and the polyreference method.[11] Overspecification of the model order introduces extraneous poles and zeros into the model. The noise bias perturbs these computational poles and zeros, reducing the perturbation of the estimated system poles and zeros. The major drawback of an order overspecified model is that it may result in an excessively large model and introduces unwanted modes. There has been no reliable method to eliminate the computational poles and zeros and reduce the model order.

2LS Algorithm

For certain applications, the full ARMA model does not have to be identified in a single step. A two-stage algorithm is proposed where the AR parameters are estimated separately using free-response data and the moving average (MA) parameters are determined from forced-response data. In the first stage, the AR parameter identification method uses overspecification to reduce bias problems and automatically eliminates the computational poles to reduce the model. The second stage then estimates a set of MA parameters using the LS algorithm and completes the ARMA model. The major disadvantage of the method is that two sets of data, free response and the input-output time histories of forced response, are required.

AR Parameter Identification

The free response is modeled as an AR process. An LS procedure, similar to the LS procedure for an ARMA model, can be developed and the natural frequencies and damping factors estimated. However, the ordinary LS estimation of the AR parameters is still sensitive to noise. The estimates of the frequencies and damping factors are improved when the model is overspecified, but it is often difficult to distinguish between the system frequencies and dampings and the extraneous ones introduced by the overspecified model. The principal eigenvectors method (PEM) for damped exponentials[12] addresses this problem. The PEM solves a set of LS normal equations through a singular value reduced-rank solution and uses the mapping in Eq. (3) to estimate the frequencies and damping factors. Because of the way the normal equations are written and solved, the system poles are distinguished from the extraneous poles. The singular values of the discrete-time autocorrelation matrix are obtained in the PEM, and the order of the system can be identified in an automated fashion.

The PEM requires expressing the AR model in the backward direction and, thus, places stable poles outside the unit circle. The overspecified model is used to increase the accuracy of the estimation in the presence of noise. The overdetermined equations for an Lth-order backward linear prediction can be written in matrix form as

$$\begin{bmatrix} y_m(2) & y_m(3) & \cdots & y_m(L+1) \\ y_m(3) & y_m(4) & \cdots & y_m(L+2) \\ \vdots & \vdots & & \vdots \\ y_m(N-L+1) & y_m(N-L+2) & \cdots & y_m(N) \end{bmatrix} \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_L \end{pmatrix} = - \begin{bmatrix} y_m(1) \\ y_m(2) \\ \vdots \\ y_m(N-L) \end{bmatrix} + \begin{bmatrix} e(1) \\ e(2) \\ \vdots \\ e(N-L) \end{bmatrix} \tag{10}$$

or

$$Q_m c = -d_m + e \tag{11}$$

The backward polynomial, the polynomial corresponding to the overspecified backward AR model, can be written as

$$C(z) = 1 + c_1 z + c_2 z^2 + \cdots + c_L z^L \tag{12}$$

There are p roots of Eq. (12) located outside the unit circle representing the poles associated with the system frequencies and damping factors. There are $L - p$ extraneous computational roots. The normal equations are

$$Q_m^T Q_m c = -Q_m^T d_m = f \tag{13}$$

A truncated singular-value decomposition (SVD) solution[12] of the normal equations forces the extraneous poles inside the unit circle, allowing the system poles to be distinguished automatically from the extraneous ones. The truncated solution is

$$c = \sum_{k=1}^{p} \sigma_k^{-1} [u_k^T f] v_k \tag{14}$$

where σ_k, u_k, and v_k are p principal eigenvalues and eigenvectors of the SVD of $Q_m^T Q_m$. Once the poles are estimated from the PEM estimate of the backward polynomial, the computational poles can be eliminated and the system poles transformed into the Laplace domain. Note that the system poles deliberately have been placed in the unstable region of the z plane, and the sign of their damping factors have to be changed to represent stable oscillations. The system frequencies and damping factors can be inferred directly from the pole locations in the Laplace domain. Reference 12 has shown that the PEM does achieve the Cramer-Rao lower bound for the variance of the frequency and damping estimates. Given the frequency and damping estimates, the inverse of Eq. (3) can be used to estimate the system pole locations in the z plane at any sample rate. A reduced-order AR model containing only the system poles is formed using these pole locations.

MA Parameter Identification

The MA parameters are determined by a modified form of the LS algorithm in the second stage of the process. Given the input-output time histories and the AR parameters, the ARMA model can be reduced to an MA model. In order to identify the MA parameters, an overdetermined set of simultaneous equations is written in matrix form as

$$\begin{bmatrix} u_m(p) & u_m(p-1) & \cdots & u_m(1) \\ u_m(p+1) & u_m(p) & \cdots & u_m(2) \\ \vdots & \vdots & & \vdots \\ u_m(N) & u_m(N-1) & \cdots & u_m(N-p+1) \end{bmatrix} \begin{pmatrix} b_0 \\ b_1 \\ \vdots \\ b_p \end{pmatrix} = \begin{pmatrix} z_m(p) \\ z_m(p+1) \\ \vdots \\ z_m(N) \end{pmatrix} + \begin{pmatrix} e(p) \\ e(p+1) \\ \vdots \\ e(N) \end{pmatrix} \tag{15}$$

or

$$U_m b = z_m + e \tag{16}$$

where

$$z_m(k) = y_m(k) - \sum_{i=1}^{p} a_i y_m(k-i) \tag{17}$$

The LS solution of Eq. (16) is

$$b = (U_m{}^T U_m)^{-1} U_m{}^T z_m \tag{18}$$

Here the concept of persistent excitation[8] is demonstrated clearly. If the scaled, discrete-time, input autocorrelation matrix, $U_m{}^T U_m$, is singular, then a unique set of MA parameters cannot be determined. A persistent excitation provides full rank of the input autocorrelation matrix.

The second stage of this algorithm solves the set of equations through the use of an LS algorithm. It should be noted that any of the major algorithms could be used to estimate the MA parameters. The MA LS parameter estimates are biased, just as the full model LS estimate is biased. Other algorithms were considered for estimation of the MA parameters, but since the LS algorithm is the simplest of the major algorithms to implement, it was chosen for this initial study.

Results

A series of experiments was performed to illustrate the application of the proposed two-stage approach. The system response time histories for the SDOF and MDOF systems were acquired experimentally from real structural vibrations and used to examine the 2LS algorithm. Results show that a model developed using single-stage LS parameter estimates (without overspecification) fails to predict response adequately for both the SDOF and MDOF systems, whereas the model developed using the 2LS algorithm parameter estimates provides very good response predictions. All measured quantities cited in the following discussion are accelerations measured in g's.

SDOF Experiment

A base motion, SDOF system was used to acquire input-output time histories to evaluate the 2LS identification procedure. The system was comprised of two masses constrained to one-dimensional movement by linear bearings and two parallel, cylindrical rods. The masses were connected by springs. The base mass could be moved manually to provide input to the system, and the response of the free mass was considered the output of the system. The system damping was provided by the bearing friction and was not viscous.

The 2LS procedure requires the free response (or impulse response data) for the AR parameter estimation in the first stage. Experimental data were acquired by constraining the base mass and displacing the free mass from equilibrium. A strain-gage accelerometer was used to measure the low-frequency acceleration response. The ordinary LS estimates for the damped natural frequency and damping factor were 1.820 Hz and 23% for a typical data set. The PEM estimates (L = 10) were 1.806 Hz and 2.0% for the same data set. The frequency estimates are similar, but the damping factors differ by an order of magnitude. A log decrement procedure was used to estimate the damping factor to be between 1.5 and 1.8%, verifying that the PEM estimate of the damping factor is more reasonable than the LS estimate. Other experimental free-response records were acquired and repeatability of the PEM estimates verified. The average damped natural frequency and damping factor from four PEM estimates were 1.805 Hz and 2.1%. The average estimates were used to produce the AR parameters used in the second stage.

Experimental input-output time histories were obtained by manual excitation of the base mass. The input to the system was measured as the acceleration of the base mass by a second strain-gauge accelerometer. Figures 1 and 2 show a typical input-output time-history set (500 points, f_s = 100 Hz); the input will be referred to as case A. Figure 3 shows another input time series, which will be referred to as case B. For case A, the input-output time-history set was used to estimate ARMA models using both the SLS and 2LS methods. Predicted responses were obtained for input case B through the use of the estimated ARMA models without knowledge of the actual input time histories. The predicted response was compared with the measured response to iput case B. Figure 4 shows the predicted response (in comparison with the actual response) for the input case B using the second-order SLS model obtained from the input-output time histories for case A. Figure 5 shows the predicted response using the second-order 2LS model obtained from the input-output time histories for case A and the AR model obtained from the first stage. The magnitude frequency-response plots for the second-order ARMA models are shown in Fig. 6. The frequency-response

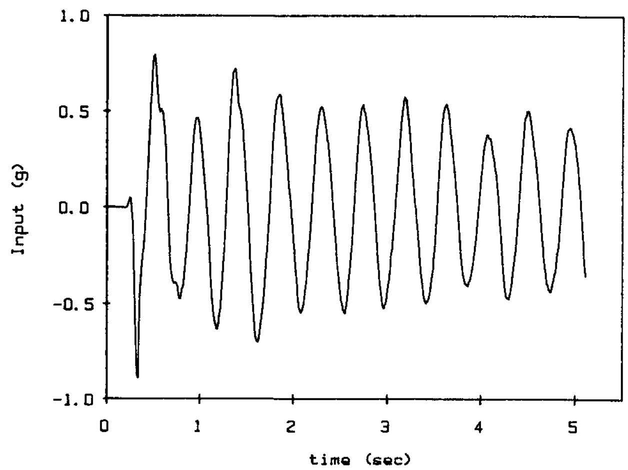

Fig. 1 Measured base acceleration, case A.

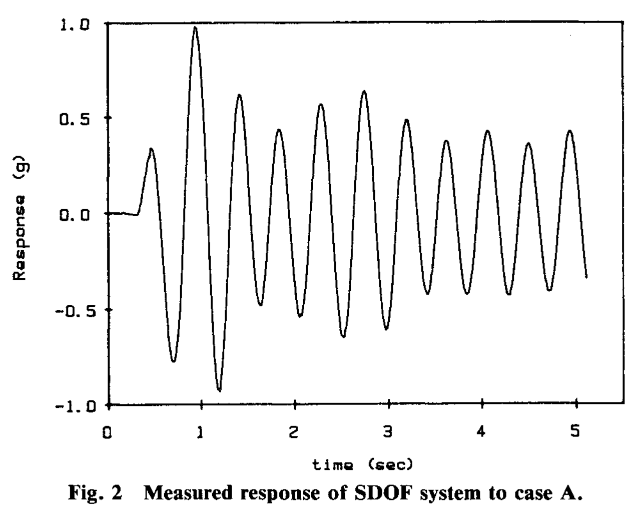

Fig. 2 Measured response of SDOF system to case A.

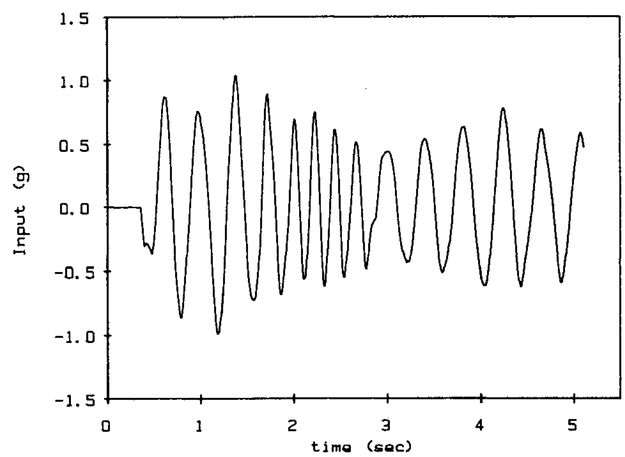

Fig. 3 Measured base acceleration, case B.

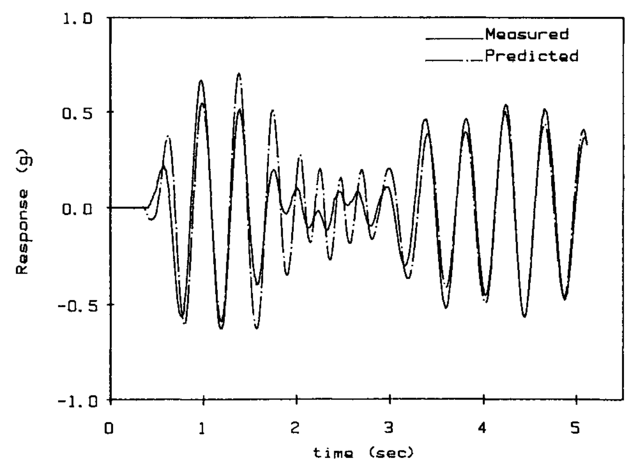

Fig. 4 Second order SLS prediction of SDOF system response to case B.

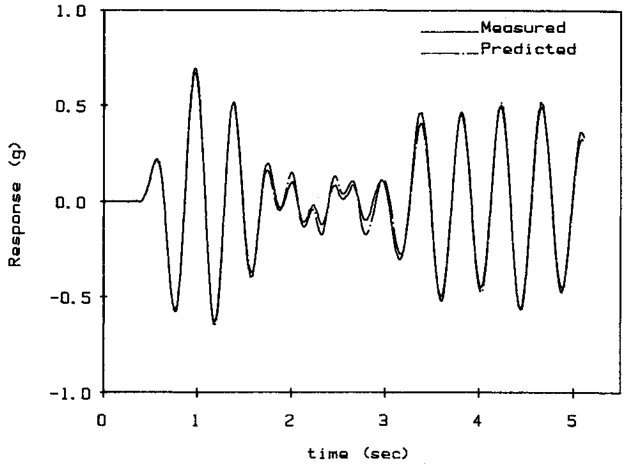

Fig. 5 Second-order 2LS prediction of SDOF response to case B.

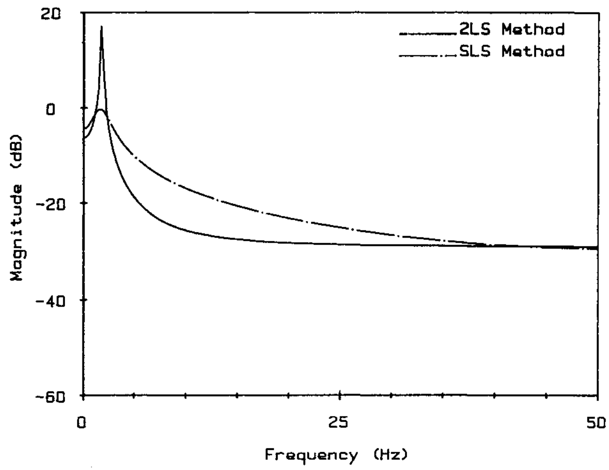

Fig. 6 Magnitude frequency response plots of second-order models.

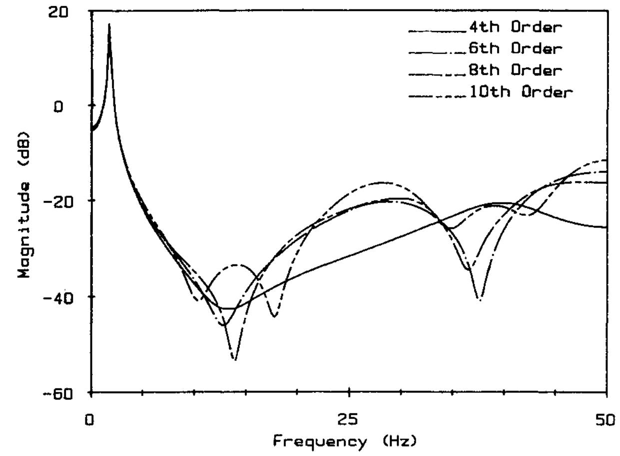

Fig. 7 Magnitude frequency response plots of overspecified SLS models.

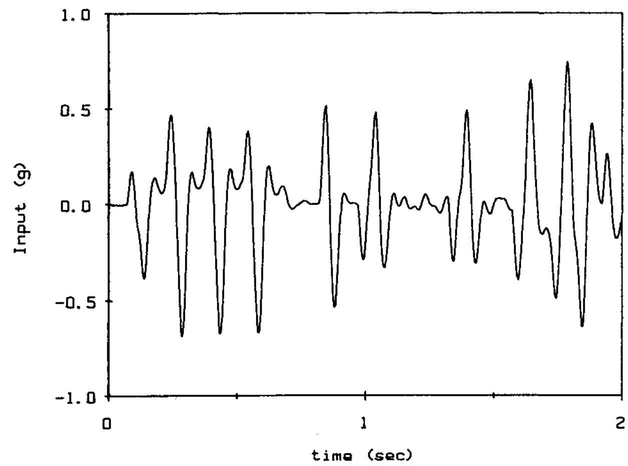

Fig. 8 Measure acceleration input to the MDOF system, case C.

Table 1 Frequency and damping estimates from single-stage LS estimates of case A

Order	Frequency, Hz	ζ,%
2	1.600	13.2
4	1.800	5.6
6	1.798	3.9
8	1.798	3.8
10	1.793	3.1
12	1.794	3.1
20	1.788	2.8

function was obtained by substituting $e^{j\pi fh}$ for z in Eq. (2) and evaluating the magnitude of the function over the desired frequency range. The SLS ARMA model has overestimated the damping compared with the 2LS ARMA model.

Overspecification of the model in the SLS procedure improves the damping estimates but introduces extraneous poles and zeros as shown in Fig. 7. The damping and frequency estimates are given in Table 1 as the model order is increased in the overspecified SLS procedure. The damping and frequency estimates using the overspecified models are not obtained automatically, a priori knowledge of the approximate frequency was used to sort the system frequency and damping estimates from the computational estimates.

MDOF Experiment

A cantilevered thin steel beam (length = 37.25 in. width = 1.5 in., thickness = 0.0625 in.) was used as an MDOF experiment system. The beam was cantilevered from a block, the block was fixed, and the beam was initially deformed to provide free vibration. A strain-gauge accelerometer was mounted on the tip of the beam to measure free response. The analog data were filtered (with cutoff frequencies of 16, 31.5, and 63 Hz) using a four-pole low-pass Butterworth filter and then digitized. The PEM algorithm was used to identify the natural frequencies and damping factors from the free-vibration response. The results appear in Table 2 along with the analytical frequency values for six different cases, two cases for each cutoff frequency setting. Note that the fourth vibrational mode was identified in trial 5 as an extraneous mode. In this case, the noise perturbed the pole locations (corresponding to the fourth mode) inside the unit cycle. The average of the identified values was then used to build an AR model for use in the second stage of the 2LS procedure.

The base block that the beam was cantilevered from was attached to a shaker to provide controllable system input. The input was measured as the acceleration response of the base block using a strain-gauge accelerometer and the system response was measured as the beam-tip acceleration. The voltage sent to the shaker was a random binary voltage generated by computer; the voltage was randomly either + 1 V or − 1 V for a discrete time. Input-output time histories of filtered accelerations are shown in Figs. 8 and 9 (500 points, f_s = 250 Hz); the input will be referred to as case c. Figure 10 shows another input referred to as case D. Most of the input energy was contained between the frequencies of approximately 5 and 35 Hz. The shaker was unable to provide input as frequencies as low as the first mode; therefore, the first mode was not excited.

The frequency-response plots of SLS ARMA models of order 4, 6, 8, and 10 obtained using input-output time histories

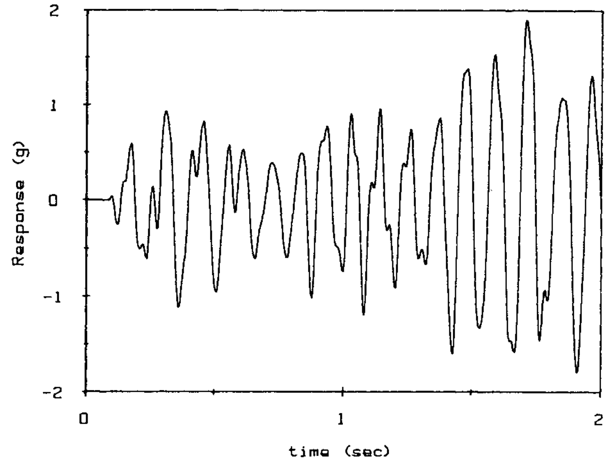

Fig. 9 Measured response of MDOF system to case C.

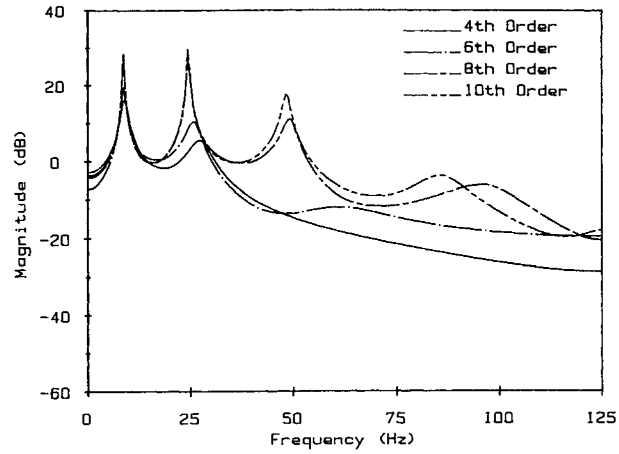

Fig. 11 Magnitude frequency response plots of SLS models of varying orders.

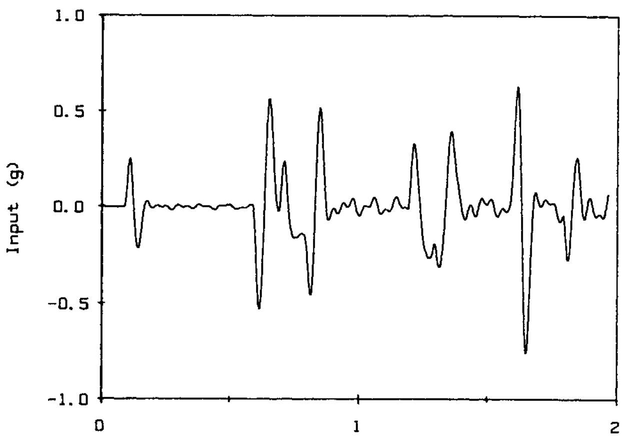

Fig. 10 Measured acceleration input to the MDOF system, case D.

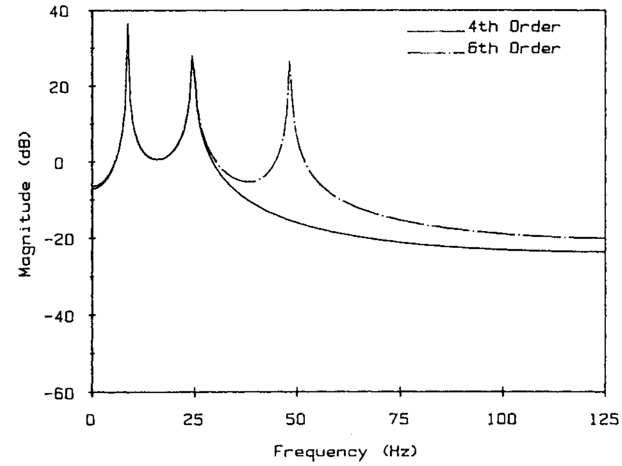

Fig. 12 Magnitude frequency response plots of 2LS models.

Table 2 PEM estimates for MDOF exp. data

Trial	L	mode 1		mode 2		mode 3		mode 4	
		freq	damp	feq	damp	freq	damp	freq	damp
#		Hz	%	Hz	%	Hz	%	Hz	%
1	30	1.516	.500	8.801	.223	(filtered)		(filtered)	
2	30	1.518	.594	8.801	.229	(filtered)		(filtered)	
3	40	1.512	.322	8.804	.205	24.66	.168	(filtered)	
4	40	1.512	.454	8.800	.271	24.64	.181	(filtered)	
5	50	1.517	.686	8.793	.307	24.57	.201	(unidentified)	
6	50	1.518	.463	8.800	.274	24.55	.330	48.29	.034
AVG		1.515	.503	8.800	.251	24.60	.220	48.29	.034
Analytical		1.45		9.10		25.49		49.95	

from case C are shown in Fig. 11. As the overspecification order is increased, the peaks are better defined. The first mode, natural frequency at approximately 1.5 Hz, was not identified by the SLS models. The 2LS method requires that the AR portion of the model be built from the frequency and damping estimates obtained from free response and the PEM. There was a question as whether to include the first mode in the calculation of the AR portion of the model. Originally the frequency and damping factors for the first mode were included, but the full ARMA models estimated in the second stage did not provide accurate predictions of the system response. Since there was no response contribution from the first mode, the frequency and damping factors for it could be excluded when the AR part of the model was constructed. When AR models of fourth and sixth order were constructed using the frequency and damping estimates of just the second and third modes (a fourth-order model) and the estimates of the second, third, and fourth modes (a sixth-order model), the resulting 2LS ARMA models were accurate predictors. The frequency-response plots for these two cases are shown in Fig. 12. The frequency-response plots show that the sixth-order model is approximately equivalent to the fourth-order model in the frequency range of interest. The predictions using the various models were compared with the actual system response. The response predictions to input case D using the sixth- and tenth-order SLS models are shown in Figs. 13 and 14. The sixth-order model does not predict the system response as well as the overspecified tenth-order model. The fourth-order 2LS model predicts the response as well as the overspecified SLS model, as shown in Fig. 15.

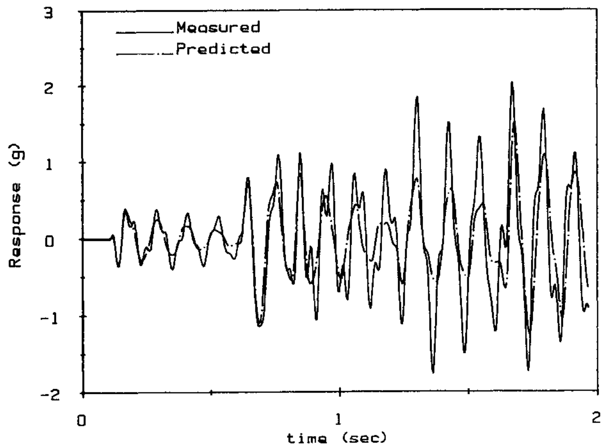

Fig. 13 Sixth-order SLS model prediction of MDOF system response to case D.

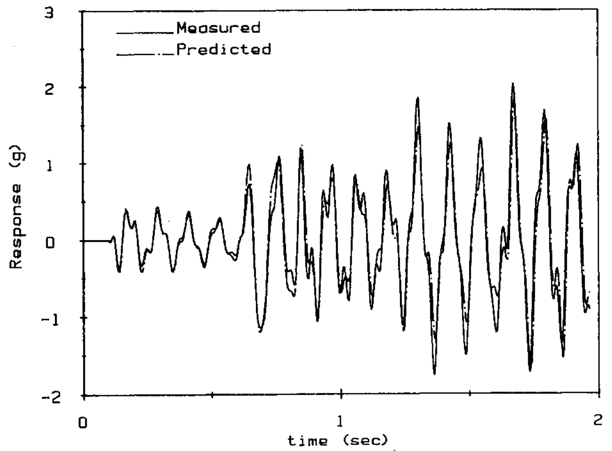

Fig. 14 Tenth-order SLS model prediction of MDOF system response to case D.

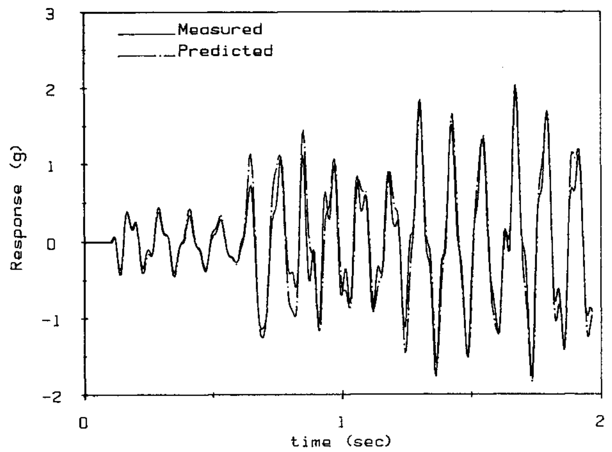

Fig. 15 Fourth-order 2LS model prediction of MDOF system response to case D.

Discussion

The ARMA model can be used to predict vibration response. If one is interested only in the vibration response at a few critical locations on a structure, it may not be necessary to build a complete modal model. A limited number of ARMA models could be identified and used to predict the system response. The difficulty with the use of the ARMA model is in the order determination and the identification of the system parameters. The deficiencies of an ordinary single-stage identification scheme arise from bias problems in the estimation of the AR parameters of the ARMA model. The advantage of the two-stage procedure is that the AR parameters are determined separately from the MA parameters, allowing greater accuracy in the estimation. A two-stage identification scheme has the additional advantage over the single-stage algorithm because it may be possible to use the same AR parameters for each ARMA model, since the natural frequencies and damping factors of the structure are not dependent on the location of the "measurement" points.

The comparison of results from the experimental systems shows the improved response predictions of the model developed using the 2LS method. The results from both systems are quite good even though both experimental systems are not viscously damped and the ARMA model is better suited for viscously damped systems. The PEM was successful in identifying the frequency and damping factors of the first four modes for the cantilever beam. The PEM did fail to provide estimates of the fourth mode of the experimental MDOF system in trial 5. In this case, the measured free response contained too much noise. The PEM sensitivity toward noise can be diminished by further overspecification of the model, but that was not attempted in this study. An overspecified ordinary LS estimate of the AR parameters could have been used to identify the frequencies and dampings, but it would have been difficult to distinguish the signal roots from the extraneous roots.

The disadvantage of the 2LS method is that it requires an additional set of data, a free-response record. Implementation of the method on practical structures requires a time-domain impulse response to be constructed from an ensemble frequency-response function for the first stage. The same time-domain data used to construct the ensemble frequency-response function can also be used in the second stage of the algorithm. In comparison the robust two-stage methods described in Refs. 2 and 3 can use the ensemble frequency-response function to construct a target spectral matrix. These methods first estimate an AR model for the spectral matrix and then convert the AR model into an ARMA model. These methods determine the parameters of the models through a minimization of frequency-domain errors. The inputs causing the system responses are unknown and must be considered white noise processes. The 2LS method described in this paper determines model parameters through a minimization of time-domain errors where the input is measured. The 2LS method is meant to be a time-domain method, but unfortunately frequency-domain averages may be required to obtain free response of a practical structure. No experimental comparison of the 2LS method and those proposed by Refs. 2 and 3 was attempted at this time.

Another problem with the 2LS method is that the second stage is an ordinary LS method. There is no means of reducing the noise bias in the estimation of the zeros for the model. This is evident from the difficulties encountered when the first mode was included in the AR model for the cantilevered beam. The second stage should cancel any unnecessary poles with corresponding zeros. When noise-free simulated data are used, the second stage does cancel the unnecessary portions of the AR model. But when noise is added, bias affects the second-stage procedure and it loses the capability to cancel out extraneous portions of the AR model.

The ideal algorithm would require only forced-response data, overspecify both the poles and the zeros, and work entirely in the time domain, eliminating the expense of building ensemble frequency-response functions. A single-stage algorithm could estimate an overspecified model, and the extraneous computational poles and roots could be eliminated, providing a small-order accurate model. The current difficulty is in the automatic sorting of the extraneous poles and roots from the signal pole and roots. The automatic sorting of the 2LS poles may provide some insight into the development of such a single-stage algorithm.

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References

[1]Hale, A. L. and Bergman, L. A., "The Dynamic Synthesis of General Non-Conservative Structures from Separately Identified Substructure Models," *Journal of Sound and Vibration,* Vol. 98, Feb. 1985, pp. 431-446.

[2]Mignolet, M. P. and Spanos, P. D. "Recursive Simulation of Stationary Multivariate Random Processes – Part I," *Transactions of the ASME,* Vol. 54, Sept. 1987, pp. 674-680.

[3]Mignolet, M. P. and Spanos, P. D., "Recursive Simulation of Stationary Multivariate Random Processes – Part II," *Transactions of the ASME,* Vol. 54, Sept. 1987, pp. 681-687.

[4]Gersch, W. and Foutch, D. A., "Least Squares Estimates of Structural System Parameters Using Covariance Function Data," *IEEE Transactions on Automatic Control,* Vol. AC-19, No. 6, Dec. 1974, pp. 898-903.

[5]Kay, S. M. and Marple, S. L., "Spectrum Analysis-A Modern Perspective," *IEEE Proceedings,* Vol. 69, No. 11, Nov. 1981, pp. 1380-1419.

[6]Akaike, H., "Canonical Correlation Analysis of Time Series and the Use of an Information Criterion," *System Identification, Advances and Case Studies,* edited by R. Mehra and D. Lainiotis, Academic Press, New York, 1976, pp. 27-96.

[7]Batill, S. M. and Hollkamp, J. J., "Time Series Modeling for Structural Response Prediction," AFWAL TR-88-3114, Wright-Patterson AFB, Dayton, OH, Nov. 1988.

[8]Åstrom, K. J. and Eykhoff, P., "System Identification – A Survey," *Automatica,* Vol. 7, March 1971, pp. 123-162.

[9]Shinozuka, M., Yun, C., and Imai, H., "Identification of Linear Structural Dynamic Systems," *Journal of the Engineering Mechanics Division,* American Society of Civil Engineers, Dec. 1982, pp. 1371-1390.

[10]Pappa, R. S. and Ibrahim, S. R., "A Parametric Study of the Ibrahim Time Domain Modal Identification Algorithm," *Shock and Vibration Bulletin,* Vol. 51, May No. 3, 1981, pp. 43-51.

[11]Vold, H., Kundrat, J., Rocklin, G., and Russell, R., "A Multi-Input Modal Estimation Algorithm for Mini-Computers," *SAE Transactions,* Vol. 91/1, Jan. 1982, pp. 815-821.

[12]Kumaresan, R. and Tufts, D. W., "Estimating the Parameters of Exponentially Damped Sinusoids and Pole-Zero Modeling in Noise," *IEEE Transactions on Acoustics, Speech, and Signal Processing,* Vol. ASSP-30, No. 6, Dec. 1982, pp. 833-840.